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## NONLEPTONIC B DECAYS AND RARE DECAYS

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### ABSTRACT

We review selected topics in the field of nonleptonic and rare  $B$  meson decays. We concentrate in particular on exclusive channels, discussing recent developments based on the concepts of factorization in QCD and the heavy-quark limit.<sup>1</sup>

### 1 Introduction

The major goal of  $B$  physics is to provide us with novel and decisive tests of the quark flavour sector. The most interesting  $B$  decay channels typically have small branching fractions below  $10^{-4}$  and are being studied by the current generation of  $B$  physics facilities. Important examples of such decays are

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nonleptonic modes as  $B \rightarrow \pi\pi$  or  $B \rightarrow \pi K$ , and the radiative rare decays  $B \rightarrow K^*\gamma$ ,  $\rho\gamma$ ,  $K^*l^+l^-$ ,  $l\nu\gamma$ . They constitute a rich source of information, in particular on CKM angles and flavour-changing neutral currents (FCNC). Many new results are becoming available from the  $B$  factories. Both inclusive and exclusive decays can be exploited. Loosely speaking, the exclusive channels are easier for experiment while they are harder for theory. The challenge for theory is to control the effects of QCD. To achieve this it is necessary to devise a systematic factorization of short-distance and long-distance contributions, which usually results in a considerable simplification of the problem. For  $B$  decay matrix elements this factorization relies on the hierarchy  $m_b \gg \Lambda_{QCD}$ . This allows us to perform an expansion around the heavy-quark limit and to factorize perturbative contributions (scales of order  $m_b$ ) from nonperturbative dynamics ( $\Lambda_{QCD}$ ). Since the general concept of factorization in QCD has recently found new applications in the important domain of *exclusive*  $B$  decays, we shall focus the following presentation on this area.

## 2 Exclusive hadronic $B$ decays in QCD

The calculation of  $B$ -decay amplitudes, such as  $B \rightarrow D\pi$ ,  $B \rightarrow \pi\pi$  or  $B \rightarrow \pi K$ , starts from an effective Hamiltonian, which has, schematically, the form

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \lambda_{CKM} C_i Q_i \quad (1)$$

Here  $C_i$  are the Wilson coefficients at a scale  $\mu \sim m_b$ ,  $Q_i$  are local, dimension-6 operators and  $\lambda_{CKM}$  represents the appropriate CKM matrix elements. The main theoretical problem is to evaluate the matrix elements of the operators  $\langle Q_i \rangle$  between the initial and final hadronic states. A typical matrix element reads  $\langle \pi\pi | (\bar{u}b)_{V-A} (\bar{d}u)_{V-A} | B \rangle$ .

These matrix elements simplify in the heavy-quark limit, where they can in general be written as the sum of two terms, each of which is factorized into hard scattering functions  $T^I$  and  $T^{II}$ , respectively, and the nonperturbative, but simpler, form factors  $F_j$  and meson light-cone distribution amplitudes  $\Phi_M$  (Fig. 1). Important elements of this approach are: i) The expansion in  $\Lambda_{QCD}/m_b \ll 1$ , consistent power counting, and the identification of the leading power contribution, for which the factorized picture can be expected to hold. ii) Light-cone dynamics, which determines for instance the properties of the fast

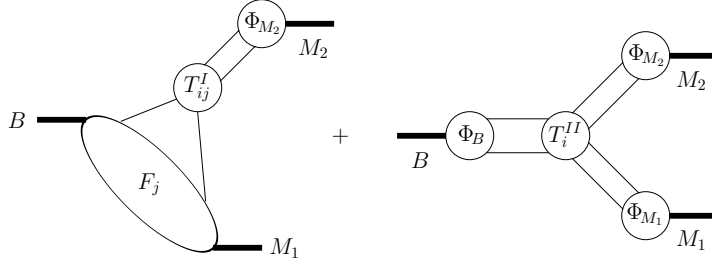


Figure 1: *Graphical representation of the factorization formula.*

light mesons. The latter are described by light-cone distribution amplitudes  $\Phi_\pi$  of their valence quarks defined as

$$\langle \pi(p) | u(0) \bar{d}(z) | 0 \rangle = \frac{i f_\pi}{4} \gamma_5 \not{p} \int_0^1 dx e^{ixpz} \Phi_\pi(x) \quad (2)$$

with  $z$  on the light cone,  $z^2 = 0$ . iii) The collinear quark-antiquark pair dominating the interactions of the highly energetic pion decouples from soft gluons (colour transparency). This is the intuitive reason behind factorization. iv) The factorized amplitude consists of hard, short-distance components, and soft, as well as collinear, long-distance contributions.

More details on the factorization formalism can be found elsewhere <sup>1)</sup>. Here we would like to emphasize an important phenomenological application. Consider the time-dependent, mixing-induced CP asymmetry in  $B \rightarrow \pi^+ \pi^-$

$$\mathcal{A}_{CP}(t) = \frac{\Gamma(B(t) \rightarrow \pi^+ \pi^-) - \Gamma(\bar{B}(t) \rightarrow \pi^+ \pi^-)}{\Gamma(B(t) \rightarrow \pi^+ \pi^-) + \Gamma(\bar{B}(t) \rightarrow \pi^+ \pi^-)} \quad (3)$$

$$= -S \sin(\Delta M_d t) + C \cos(\Delta M_d t) \quad (4)$$

Using CKM-matrix unitarity, the decay amplitude consists of two components with different CKM factors and different hadronic parts, schematically

$$A(B \rightarrow \pi^+ \pi^-) = V_{ub}^* V_{ud}(\text{up} - \text{top}) + V_{cb}^* V_{cd}(\text{charm} - \text{top}) \quad (5)$$

If the penguin contribution  $\sim V_{cb}^* V_{cd}$  could be neglected, one would have  $C = 0$  and  $S = \sin 2\alpha$ , hence a direct relation of  $\mathcal{A}_{CP}$  to the CKM angle  $\alpha$ . In reality the penguin contribution is not negligible compared to the dominant

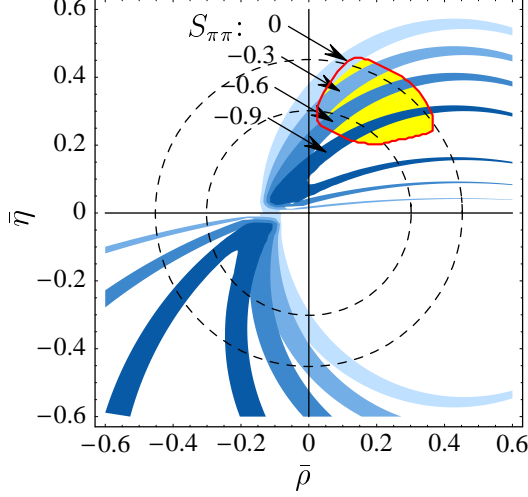


Figure 2: Constraints in the  $\bar{\rho}$ ,  $\bar{\eta}$  plane from CP violation observable  $S$  in  $B \rightarrow \pi^+\pi^-$ . The constraints from  $|V_{ub}/V_{cb}|$  (dashed circles) and from the standard analysis of the unitarity triangle (irregular shaded area) are also shown.

tree contribution  $\sim V_{ub}^* V_{ud}$ . The ratio of penguin and tree amplitude, which enters the CP asymmetry, depends on hadronic physics. This complicates the relation of observables  $S$  and  $C$  to CKM parameters. QCD factorization of  $B$ -decay matrix elements allows us to compute the required hadronic input and to determine the constraint in the  $(\bar{\rho}, \bar{\eta})$  plane implied by measurements of the CP asymmetry. This is illustrated for  $S$  in Fig. 2. The widths of the bands indicate the theoretical uncertainty<sup>2)</sup>. Note that the constraints from  $S$  are relatively insensitive to theoretical or experimental uncertainties. The analysis of direct CP violation measured by  $C$  is more complicated due to the importance of strong phases. The current experimental results are

$$\begin{aligned} C &= -0.94^{+0.31}_{-0.25} \pm 0.09 \quad (\text{Belle}) & -0.30 \pm 0.25 \pm 0.04 \quad (\text{Babar}) \\ S &= -1.21^{+0.38+0.16}_{-0.27-0.13} \quad (\text{Belle}) & +0.02 \pm 0.34 \pm 0.05 \quad (\text{Babar}) \end{aligned} \quad (6)$$

QCD factorization to leading power in  $\Lambda/m_b$  has been demonstrated at  $\mathcal{O}(\alpha_s)$  for the important class of decays  $B \rightarrow \pi\pi, \pi K$ . For  $B \rightarrow D\pi$  (class I), where hard spectator interactions are absent, a proof has been given explicitly at two loops<sup>1)</sup> and to all orders in the framework of soft-collinear effective

theory (SCET) <sup>3)</sup>. Complete matrix elements are available at  $\mathcal{O}(\alpha_s)$  (NLO) for  $B \rightarrow \pi\pi, \pi K$ , including electroweak penguins. Power corrections are presently not calculable in general. Their impact has to be estimated and included into the error analysis. Critical issues here are annihilation contributions and certain corrections proportional to  $m_\pi^2/((m_u + m_d)m_b)$ , which is numerically sizable, even if it is power suppressed. However, the large variety of channels available will provide us with important cross checks and arguments based on SU(2) or SU(3) flavour symmetries can also be of use in further controlling uncertainties.

### 3 Radiative decays $B \rightarrow V\gamma$

Factorization in the sense of QCD can also be applied to the exclusive radiative decays  $B \rightarrow V\gamma$  ( $V = K^*, \rho$ ). The factorization formula for the operators in the effective weak Hamiltonian can be written as <sup>4, 5)</sup>

$$\langle V\gamma(\epsilon)|Q_i|\bar{B}\rangle = \left[ F^{B\rightarrow V}(0) T_i^I + \int_0^1 d\xi dv T_i^{II}(\xi, v) \Phi_B(\xi) \Phi_V(v) \right] \cdot \epsilon \quad (7)$$

where  $\epsilon$  is the photon polarization 4-vector. Here  $F^{B\rightarrow V}$  is a  $B \rightarrow V$  transition form factor, and  $\Phi_B, \Phi_V$  are leading twist light-cone distribution amplitudes (LCDA) of the  $B$  meson and the vector meson  $V$ , respectively. These quantities describe the long-distance dynamics of the matrix elements, which is factorized from the perturbative, short-distance interactions expressed in the hard-scattering kernels  $T_i^I$  and  $T_i^{II}$ . The QCD factorization formula (7) holds up to corrections of relative order  $\Lambda_{QCD}/m_b$ . Annihilation topologies are power-suppressed, but still calculable in some cases. The framework of QCD factorization is necessary to compute exclusive  $B \rightarrow V\gamma$  decays systematically beyond the leading logarithmic approximation. Results to next-to-leading order in QCD, based on the heavy quark limit  $m_b \gg \Lambda_{QCD}$  have been computed <sup>4, 5)</sup> (see also <sup>6)</sup>).

The method defines a systematic, model-independent framework for  $B \rightarrow V\gamma$ . An important conceptual aspect of this analysis is the interpretation of loop contributions with charm and up quarks, which come from leading operators in the effective weak Hamiltonian. These effects are calculable in terms of perturbative hard-scattering functions and universal meson light-cone distribution amplitudes. They are  $\mathcal{O}(\alpha_s)$  corrections, but are leading power contributions in the framework of QCD factorization. This picture is in contrast to

the common notion that considers charm and up-quark loop effects as generic, uncalculable long-distance contributions. Non-factorizable long-distance corrections may still exist, but they are power-suppressed. The improved theoretical understanding of  $B \rightarrow V\gamma$  decays strengthens the motivation for still more detailed experimental investigations, which will contribute significantly to our knowledge of the flavour sector.

The uncertainty of the branching fractions is currently dominated by the form factors  $F_{K^*}$ ,  $F_\rho$ . A NLO analysis <sup>5)</sup> yields (in comparison with the experimental results in brackets)  $B(\bar{B} \rightarrow \bar{K}^{*0}\gamma)/10^{-5} = 7.1 \pm 2.5$  ( $4.21 \pm 0.29$  <sup>7)</sup>) and  $B(B^- \rightarrow \rho^-\gamma)/10^{-6} = 1.6 \pm 0.6$  ( $< 2.3$  <sup>8)</sup>). Taking the sizable uncertainties into account, the results for  $B \rightarrow K^*\gamma$  are compatible with the experimental measurements, even though the central theoretical values appear to be somewhat high.  $B(B \rightarrow \rho\gamma)$  is a sensitive measure of CKM quantities such as the angle  $\gamma$ .

#### 4 Forward-backward asymmetry zero in $B \rightarrow K^*l^+l^-$

Substantial progress has taken place over the last few years in understanding the QCD dynamics of exclusive  $B$  decays. The example of the forward-backward asymmetry in  $B \rightarrow K^*l^+l^-$  nicely illustrates some aspects of these developments.

The forward-backward asymmetry  $A_{FB}$  is the rate difference between forward ( $0 < \theta < \pi/2$ ) and backward ( $\pi/2 < \theta < \pi$ ) going  $l^+$ , normalized by the sum, where  $\theta$  is the angle between the  $l^+$  and  $B$  momenta in the centre-of-mass frame of the dilepton pair.  $A_{FB}$  is usually considered as a function of the dilepton mass  $q^2$ . In the standard model the spectrum  $dA_{FB}/dq^2$  (Fig. 3) has a characteristic zero at

$$\frac{q_0^2}{m_B^2} = -\alpha_+ \frac{m_b C_7}{m_B C_9^{eff}} \quad (8)$$

depending on short-distance physics contained in the coefficients  $C_7$  and  $C_9^{eff}$ . The factor  $\alpha_+$ , on the other hand, is a hadronic quantity containing ratios of form factors.

It was first stressed in <sup>9)</sup> that  $\alpha_+$  is not very much affected by hadronic uncertainties and very similar in different models for form factors with  $\alpha_+ \approx 2$ . After relations were found between different heavy-light form factors ( $B \rightarrow P$ ,

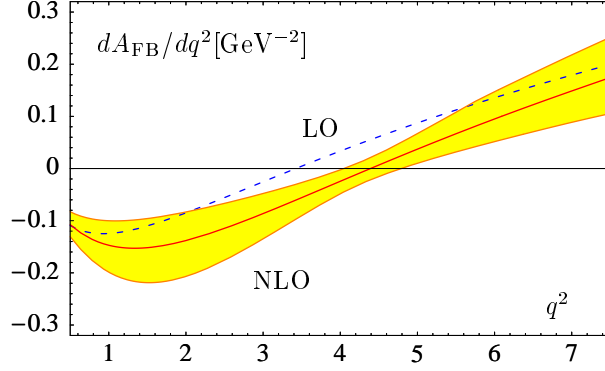


Figure 3:  $A_{FB}$  spectrum for  $\bar{B} \rightarrow K^* l^+ l^-$  at leading and next-to-leading order in QCD (from <sup>4)</sup>).

$V$ ) in the heavy-quark limit and at large recoil <sup>10)</sup>, it was pointed out in <sup>11)</sup> that as a consequence  $\alpha_+ = 2$  holds exactly in this limit. Subsequently, the results of <sup>10)</sup> were demonstrated to be valid beyond tree level <sup>4, 12)</sup>. The use of the  $A_{FB}$ -zero as a *clean* test of standard model flavour physics was thus put on a firm basis and NLO corrections to (8) could be computed <sup>4)</sup>. More recently also the problem of power corrections to heavy-light form factors at large recoil in the heavy-quark limit has been studied <sup>13)</sup>. Besides the value of  $q_0^2$ , also the sign of the slope of  $dA_{FB}(\bar{B})/dq^2$  can be used as a probe of new physics. For a  $\bar{B}$  meson, this slope is predicted to be positive in the standard model <sup>14)</sup>.

## 5 Radiative leptonic decay $B \rightarrow l\nu\gamma$

The tree-level process  $B \rightarrow l\nu\gamma$  is not so much of direct interest for flavour physics, but it provides us with an important laboratory for studying QCD dynamics in exclusive  $B$  decays, which is crucial for many other applications. The leading-power contribution comes from the diagram in Fig. 4 (b), which contains a light-quark propagator that is off-shell by an amount  $(q-k)^2 \sim q_- k_+$ . Here  $q$  is the hard, light-like momentum of the photon with components scaling as  $m_b$  (this restricts the region of phase-space where the present discussion

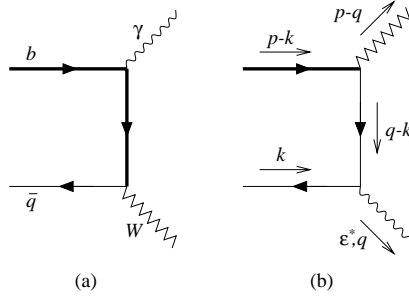


Figure 4: *Tree-level diagrams for  $B \rightarrow l\nu\gamma$ . Only diagram (b) contributes at leading power (see [16]).*

applies), and  $k$  is the soft momentum of the spectator quark. The decay is thus determined by a hard-scattering process, but also depends on the structure of the  $B$  meson in a non-trivial way [15]. Recently, in [16] it has been proposed, and shown to one loop in QCD, that the form factors  $F$  for this decay factorize as

$$F = \int d\tilde{k}_+ \Phi_B(\tilde{k}_+) T(\tilde{k}_+) \quad (9)$$

where  $T$  is the hard-scattering kernel and  $\Phi_B$  the light-cone distribution amplitude of the  $B$  meson defined as

$$\Phi_B(\tilde{k}_+) = \int dz_- e^{i\tilde{k}_+ z_-} \langle 0 | b(0) \bar{u}(z) | B \rangle |_{z_+ = z_\perp = 0} \quad (10)$$

The hard process is characterized by a scale  $\mu_F \sim \sqrt{m_b \Lambda}$ . At lowest order the form factors are proportional to  $\int d\tilde{k}_+ \Phi_B(\tilde{k}_+) / \tilde{k}_+ \equiv 1/\lambda_B$ , a parameter that enters hard-spectator processes in many other applications. The analysis at NLO requires resummation of large logarithms  $\ln(m_b/\tilde{k}_+)$ . An extension of the proof of factorization to all orders was subsequently given by [17] within the SCET.

## 6 Conclusions

Factorization formulas in the heavy-quark limit have been proposed for a large variety of exclusive  $B$  decays. They justify in many cases the phenomenological factorization ansatz that has been employed in many applications. In addition they enable consistent and systematic calculations of corrections in powers of



$\alpha_s$ . Non-factorizable long-distance effects are not calculable in general but they are suppressed by powers of  $\Lambda_{QCD}/m_b$ . So far,  $B \rightarrow D^+\pi^-$  decays are probably understood best. Decays with only light hadrons in the final state such as  $B \rightarrow \pi\pi$ ,  $K^*\gamma$ ,  $\rho\gamma$ , or  $K^*l^+l^-$  include hard spectator interactions at leading power and are therefore more complicated. An important new tool that has been developed is the soft-collinear effective theory (SCET), which is of use for proofs of factorization and for the theory of heavy-to-light form factors at large recoil. Recent studies of the prototype process  $B \rightarrow l\nu\gamma$  have also led to a better understanding of QCD dynamics in exclusive hadronic  $B$  decays. All these are promising steps towards achieving a good theoretical control over QCD dynamics in rare hadronic  $B$  decays, which is necessary for probing CP violation, flavour physics and new phenomena at short distances.

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